Société de Calcul Mathématique SA
Tools for decision help

Fédération Française des Jeux Mathématiques

Mathematical Competitive Game 2015-2016

False Alarms in a Sensor Network

Fédération Française des Jeux Mathématiques
(French Federation of Mathematical Games)

and

Société de Calcul Mathématique SA

in partnership with

IRSN

(Institut de Radioprotection et de Sûreté Nucléaire)

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I. Presentation of the Games

The "Mathematical Games", jointly organized by FFJM and SCM, have existed for six years; the previous ones were:

- In 2008-2009, conception of a bus transportation network in a city, in partnership with Veolia Transport;
- In 2009-2010, conception of an electricity distribution network, in partnership with RTE (Réseau de Transport d'Electricité);
- In 2011-2012, search for the best itinerary by a car, in partnership with the newspaper Auto Plus;
- In 2012-2013, fighting forest fires in Siberia, in partnership with the Paris Firemen Brigade;
- In 2013-2014, checking an industrial process;
- In 2014-2015, uncertainties in GPS positioning, in partnership with the French Institute for Transportation Science and Technology, Geolocalisation Team (IFSTTAR/-CoSys/Geoloc) and The French Ministry of Transportation, Mission for Tarification Pricing (MEDDTL/DIGITIM/SAGS/MT).

They deal with the resolution of a "real life" problem, that is a problem of general concern, but simplified in its mathematical contents. Still, the resolution typically requires several months of work.

Candidates may compete individually or as groups, for instance high school classes, or college students, or university students, preparing a "memoir" for the end of their studies.

Two categories of prizes are given:

Individual prizes:
For the winner: 500 Euros
For the second: 200 Euros
For the next three: 100 Euros each.

Prizes for groups:
For the winner: 500 Euros
For the second: 200 Euros
For the next three: 100 Euros each.

The total amount of prizes is therefore 2 000 Euros. The best solutions are published on the web site of FFJM, on the web site of SCM, and on the web sites of our partners. The official announcement of the results and the ceremony for prizes occur during the "Salon de la Culture et des Jeux Mathématiques" (Fair for Mathematical Culture and Games), which is held in Paris, each year, during the month of May.

The winners, previous years, gained considerable notoriety, both in the press and television in their respective countries.

II. The 2015-2016 Prize

A. General presentation of the subject

Everyone, nowadays, has heard about "surveillance networks" : they usually consist in a set of sensors, which are supposed to detect a danger and send an alert. For instance, there are detection stations in regions exposed to fire hazards (see our 2012-2013 Game : Fighting Fires in Siberia). Also, stations may detect a pollution in the air, in the water, and so on, and send a signal. Another typical example is the one of sensors detecting dangerous gases in coal mines.

In 2007, a disaster occured in Zasyadko, Donetsk region, and more than 100 miners were killed by a methane explosion. It was thought that the network of sensors did not work correctly. SCM was consulted by the local authorities: could we improve the safety of the network ? Our answer was that each sensor must be monitored permanently, using a comparison with its neighbors. So one has to design a very dense network of sensors. And when this is done, double it, and when this is done, double it again. This is a vital piece of equipment, and when people are 1,000 meters below the surface, there is no room for mathematical optimisation.

In general, to each detection station is associated a "region", which is the set "seen" by the station. This region is often a circle (or more exactly a disk), if the station sees equally in all directions (this is the case for fire detections). In other circumstances, it may be modified, for instance if one takes the winds into account (air pollution). In a river, each station will cover a segment, upstream of the station.

No matter what the shape of the region is, people usually consider that the fewer stations, the better, because these stations are costly to buy, to install, to verify, and if we have fewer measurements, it will be easier to treat them. So, the question of the covering of the territory by the stations usually reduces to a minimal covering problem. If each region is a disk, we would determine the smallest number of disks covering the territory.

What the present Game shows is that this approach is completely wrong if one does not take into account (as one should !) the possibility of false alarms. False alarms do exist in real life settings, and they are always neglected in the definition of a network of sensors. So the result is not correct in general.

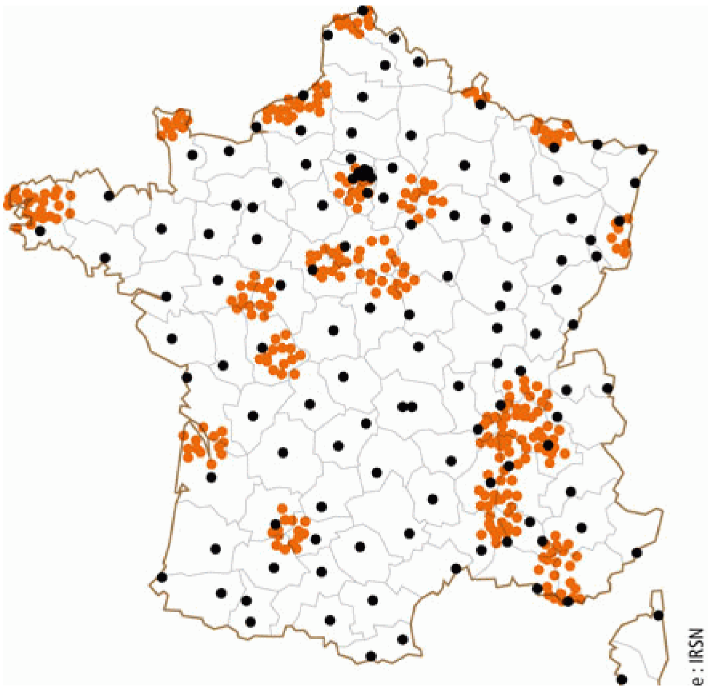
One should take into account, also, the possibility of failure of each equipment. But, in the case which will occupy us here, the failure has a very small probability, as we will see below.

B. The existing situation

It deals with a network of stations, which monitor the radioactivity in the air, and send an alert if this radioactivity is above a certain threshold. This network has existed in France since 1991 ; its name is TELERAY, and it is operated by the French Institut de Radioprotection et de Sûreté Nucléaire (IRSN).

See http://www.irsn.fr/FR/connaissances/Environnement/Pages/Home.aspx (in French).

Here is a map of France with the existing stations (the black points are newer ones).



Similar networks exist in Germany and in other countries.

As one sees, the existing network obeys a simple logic: the old stations were concentrated near the nuclear plants; the new ones are put in each French "département" (an administrative division), near the main city of this "département" (France, mainland, is divided into 96 "départements").

C. The present problem

We will not look at the TELERAY network any longer; we will instead consider a simplified territory (about the same size as France, but much simpler in geometry), try to list the threats, and try to come up with a solution: where should the sensors be ?

1. Basic data

The basic data are:

Domain : a square of 750 km x 750 km, that is 562 500 km2 (continental France has an area of 550 000 km2). The coordinates will be from 1 to 750 on the x axis, from 1 to 750 on the y axis. Each square kilometer will be considered as a "pixel".

We have 20 nuclear plants, the coordinates of which are:

X coord	295	490	694	11	662	473	668	422	361	409
Y coord	655	704	699	15	329	433	480	177	262	442

X coord	92	633	60	345	265	294	508	281	438	505
Y coord	565	34	195	452	651	196	129	461	737	53

Here is the position of nuclear plants:

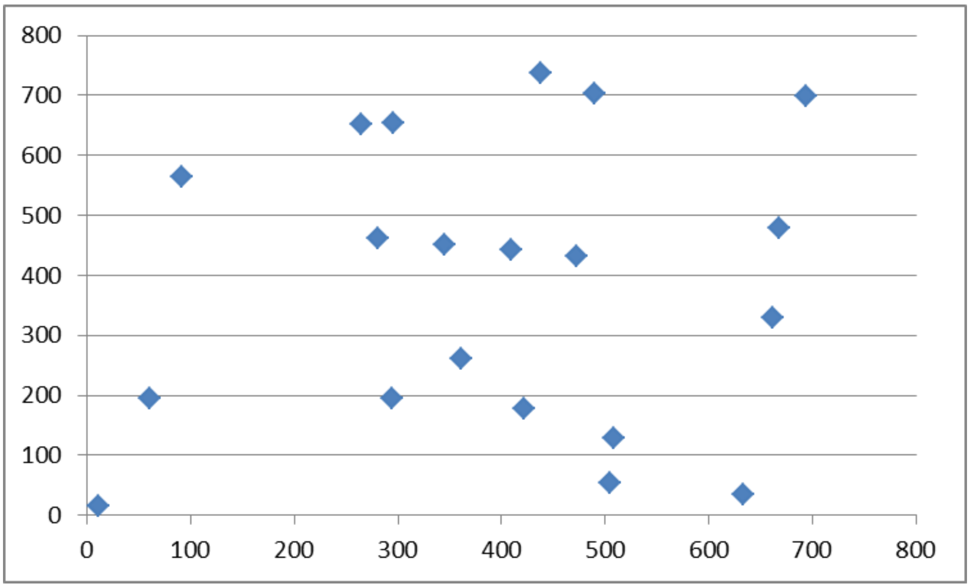


Figure 1 : the position of nuclear plants

2. The threats

There are three types of threat :

1. Each nuclear plant is considered as a possible danger, so we have to put one sensor in the same pixel. This is just a precaution, because the nuclear plants are closely monitored, permanently, by EDF, the French electricity.

2. The second threat is as follows: we worry about a possible "cloud of radioactivity", coming from the East, because there might be some nuclear accident in a country situated East of France. So we want to put a certain number of sensors on our Eastern boarder (which represents 750 pixels).

3. Finally, we have a "diffuse" threat, saying that there might be a terrorist attack anywhere on our territory: a kind of "dirty bomb", which releases radionuclides in the environment, but is not a nuclear bomb. This kind of bomb causes very little damage, but people overreact to such a threat.

3. Probabilities and costs of accidents

For a nuclear plant, the probability of a radioactive leak in the atmosphere is estimated at 5×10^{-5} per year, for each plant. All plants may reasonably considered as independent. The estimated cost of an accident is 10^{9} Euros.

The probability of an accident in some Eastern country, which would affect our eastern border is estimated at 10^{-4} per year. We do not know where the accident would happen and what point of our border would be affected. The estimated cost is 10^{5} Euros.

The probability of a terrorist action, concerning any particular pixel is 10^{-5} per year, for the whole country, to be converted to any pixel. The estimated cost is 10^{6} Euros.

Of course, all probabilities and costs are rough estimates.

Information about radioactive clouds

We will admit that a radioactive cloud takes the shape of a disk ; the influence of the winds on this shape is neglected. If the cloud originates from a nuclear plant or from a terrorist attack (threats 1 and 3), it starts with a very small disk, which expands ; in the case of a cloud coming from the East (threat 2), the size of the cloud when it enters France is unknown. In all cases, the speed and direction of winds (therefore the speed and direction of movement for the cloud) are unknown.

4. Information about sensors

We have 400 sensors to be installed; they can be installed in any particular pixel. Each sensor detects only locally: it detects the cloud only if the cloud touches the sensor.

Each sensor acts with a 0 or 1 rule: either it says nothing, or it says "1", which means "alert". Each sensor works continuously, but, in order to simplify matters, we will admit that it sends its signal each hour.

The probability that a given sensor does not work at all is 10^{-6} per year, for each sensor ; all sensors are assumed to be independent, and independent of the accidents. Since the accidents have themselves a very low probability, we see that the probability that a sensor misses something which really exists is extremely low, to be neglected in practice.

A false alarm occurs when a sensor sends "1" when there is nothing: we admit that this is the case on average twice a year, for each sensor.

5. Benefits

Of course, a sensor network does not prevent an accident, but it can diminish its consequences, since the populations receive an alert.

We admit the following :

- For a nuclear accident in a plant, if the sensor works correctly, the benefit might be to save one half of the estimated cost, that is 0.5×10^9 Euros.

- For an accident detected on the East border, the benefit might be one tenth of the estimated cost, that is 10^4 Euros.

- For a terrorist attack at any point, since it would allow the evacuation from this point, the benefit might be two thirds of the estimated cost, that is 0.66×10^6 Euros.

6. Costs of the network

First, there is a constant maintenance cost, evaluated to 5×10^3 Euros per year and per sensor.

Next, there is "false alarm cost" : if a sensor gives an alert which does not correspond to any reality, the cost is estimated to 10^5 Euros (corresponding to the mobilisation of resources, such as police and specific services, psychological impact upon population, technicians' visits to replace the sensor, and so on).

One should distinguish carefully between malfunction and false alarm :

- If a sensor is isolated and gives an alert, this alert is always taken seriously, so it may lead to a false alarm ;

- On the other hand, if a sensor is close enough to another sensor, and if both give different indications, the authorities will be suspicious and check before giving the alert. The cost of a false alarm which has been detected soon enough is only 10^3 Euros.

III. Questions

- Where should we put the 400 sensors in order to optimize the benefit of the system ?
- What is this benefit (algebraic difference between what it brings and what it costs) ?
- What would be the results if we had 800 sensors instead of 400 ?

IV. Participation rules

The game starts on November 1st, 2015 and ends on April 30th, 2016. Prizes will be given in May 2016, during the "Salon des Jeux Mathématiques", in Paris.

Participants should send their solution, in pdf format, in English or in French, no later than April 30th, 2016, to the email address: **ffjm@wanadoo.fr.**

No preliminary registration is required. Everyone can participate.