Société de Calcul Mathématique SA
Outils d'aide à la décision

Fédération Française des
Jeux Mathématiques

Mathematical Competitive Game 2018-2019

Traffic Jams in Houston, Texas

Fédération Française des Jeux Mathématiques
(French Federation of Mathematical Games)

and

Société de Calcul Mathématique SA
(Mathematical Modelling Company, Corp.)

redaction: Bernard Beauzamy and Gottfried Berton, SCM SA

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I. Presentation of the Games

The "Mathematical Games", jointly organized by FFJM and SCM, have existed for nine years; the previous ones were:

- In 2008-2009, Conception of a bus transportation network in a city, in partnership with Veolia Transport;
- In 2009-2010, Conception of an electricity distribution network, in partnership with RTE (Réseau de Transport d'Electricité);
- In 2011-2012, Search for the best itinerary by a car, in partnership with the newspaper Auto Plus;
- In 2012-2013, Fighting forest fires in Siberia, in partnership with the Paris Firemen Brigade;
- In 2013-2014, Checking an industrial process;
- In 2014-2015, Uncertainties in GPS Positioning, in partnership with the French Institute for Transportation Science and Technology, Geolocalisation Team (IFSTTAR/CoSys/Geoloc) and the French Ministry of Transportation, Mission for Tarification Pricing (MEDDTL/DIGITIM/SAGS/MT);
- In 2015-2016, False Alarms in a Sensor Network, in partnership with the French IRSN (Institut de Radioprotection et de Sûreté Nucléaire);
- In 2016-2017, From the Earth to the Moon;
- In 2017-2018, Distribution of Goods.

They deal with the resolution of a "real life" problem, that is a problem of general concern, but simplified in its mathematical contents. Still, the resolution typically requires several months of work.

Candidates may compete individually or as groups, for instance high school classes, or college students, or university students, preparing a "memoir" for the end of their studies.

Two categories of prizes are given:

Individual prizes:
For the winner: 500 Euros
For the second: 200 Euros
For the next three: 100 Euros each.

Prizes for groups:
For the winner: 500 Euros
For the second: 200 Euros
For the next three: 100 Euros each.

The total amount of prizes is therefore 2 000 Euros. The best solutions are published on the web site of FFJM, on the web site of SCM, and on the web sites of our partners. The official announcement of the results and the ceremony for prizes occur during the "Salon de la Culture et des Jeux Mathématiques" (Fair for Mathematical Culture and Games), which is held in Paris, each year, during the month of May.

The winners, previous years, gained considerable recognition, both in the press and television in their respective countries.

II. The 2018-2019 Game

A. General presentation of the subject

All major cities, anywhere on Earth, experience traffic jams, and this is not new (they existed in Paris during the Middle Ages!). However, the users are more and more in a hurry, time is more and more considered as a value, and planification is more and more in demand.

If one wants to reduce traffic jams, there are only two possibilities:

- Reduce or modify the traffic;
- Improve the physical network of routes (in short PNR).

Both may be combined, of course.

Reduce or modify the traffic may have various meanings: ask people not to use their cars, use detours, slow down some incoming traffic, use traffic lights, use tolls, parking limitations, and so on.

Improve the physical network of routes means enlarge the ways, remove obstacles, add more lanes, more tunnels, more bridges, build new routes, and so on.

In general, in Europe, most cities try to act upon the traffic (for instance asking people to use bicycles instead of cars), but they underestimate the influence of such decisions upon the economic activity. The result of a limitation in the freedom of use, concerning cars, is always a slowdown in economic activity. But, of course, a modification of the PNR, which is "hardware", is usually more costly.

B. A Global Optimization: Not a Good Idea!

The PNR being as it is, one may think of a global optimization, under the following ideas. One knows, at least roughly, the point of departure and the point of arrival for each car, and the problem is to assign to each car an "optimal route", so as to minimize the total amount of time for all trips. This approach, mathematically oriented, seems satisfactory. It is in fact entirely wrong, for two reasons:

– First reason: One never knows exactly the demand (people who want to go from point A to point B); it varies with time, with weather, with special events, and so on, and any mathematical optimization is "fragile", meaning that the result is extremely sensitive to the choice of initial parameters. The global solution may be completely different, under very small differences in initial conditions (and the users do not understand this).

– Second reason: It means that the City will give orders to the drivers (those living in this area must go through this route), a situation that the users do not accept.

C. The proposed approach

The input of the Game is a map of Houston, reduced, for simplicity, to the highway system. We consider an ordinary week day, at a peak traffic hour: on January 29th 2018 between 8 a.m. and 9 a.m. We know the number of cars which enter the highway system, at each point, during this period of time, and also the number of cars which leave it. We assume that the number of cars present on the system at 8 a.m. and at 9 a.m. are the same. Please refer to the file:
http://www.scmsa.eu/archives/SCM_FFJM_Information_highways_2018_2019.xls

Fig 1: simplified map; a more detailed map is given in the Appendix

We also know the characteristics of each highway: number of lanes, speed limits, and so on.

So, from these data, one can observe traffic jams at some places. These jams are apparent from a computation (for instance the number of lanes is insufficient for the incoming traffic); they may be supported by observations (real-time maps of traffic are available on the internet). We take into account only structural limitations of the PNR, not short term limitations, such as the ones resulting from a car accident, a flooding, roadworks, and so on.

Phase 1 of the Game is a description (with data) of the present situation in Houston in terms of flux and times: how long does it take to go from point A to point B, and how many cars do this?

Phase 2 is a list of proposed modifications to the PNR; the participants to the Game may decide to plan any modification of the PNR they wish: for instance narrowing/enlarging a portion of a route, adding/subtracting more lanes, changing a speed limit, adding/subtracting more entrances to the freeways or more exits, and even adding more freeways (for instance, what would be the influence upon traffic if one more freeway was built between Pasadena and Pearland ?). Then, they should indicate how much these modifications will cost and what benefit will result.

D. Mathematical approach

1. Maximal Flux of Vehicles

The key concept here will be the "maximal flux of vehicles" (in short MFV), measured in number of vehicles going through some line during one hour. See picture below.

The flux of vehicles is defined for any lane, and two parallel lanes may not have the same flux. For instance, in this highway, the left lane may accommodate 2000 veh/hour and the right lane only 1500.

The maximal flux is assumed to be constant over our 1 hour interval of time, which means that if the MFV is 2000 v/h, in five minutes it will be $\frac{2000}{12}$ vehicles.

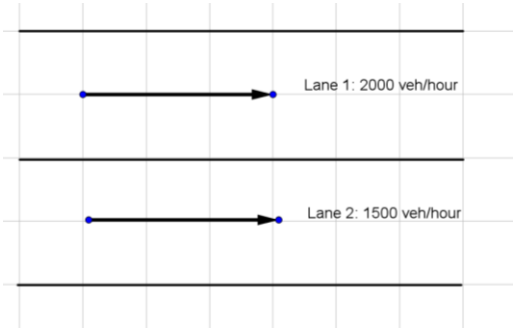

Fig2: two different lanes with two different flux

The speed limit is known at any point of the highway system (see attached document, reference above); this speed limit is related to the MFV the following way: each vehicle has its own length (say 5 meters, to be generous), plus some safety interval between two consecutive vehicles. The length of the safety interval depends itself on the speed (it should be computed so that the second vehicle has time enough to stop, should the previous one decide to do so). The participants should calculate themselves the safety interval and deduce the MFV. For the buses, the length of the vehicle will be 10 meters and 15 meters for the trucks.

The speed limit, in order to make a simplification, will be assumed to be the same for cars, buses and trucks. The proportion of trucks, buses, among all vehicles is left to the participants.

The participants will calculate the MFV in function of the speed limit, the number of lanes, and the safety interval. This will allow them to find the sections of the highway system where the incoming traffic is too important compared to the maximum flux that the infrastructure can accommodate.

2. Incoming traffic

The maximal flux of vehicles is a physical property of each lane; it means that the lane cannot accommodate more vehicles. The value of the incoming traffic at any point of the highway system is given on the attached document. This traffic corresponds to the value of the present situation in Houston, on January 29th 2018 at 8 a.m. This is an average traffic on the period from 8 a.m. to 9 a.m.: an incoming traffic of 15 000 vehicles per hour means that between 8 a.m. and 9 a.m., 15 000 vehicles have entered in the system.

So, if the number of incoming vehicles is too high, they will have to "queue", which means that a traffic jam will occur. For instance, if the MFV is 1200 veh/hour (that is 20 per minute), and if 30 per minute want to pass, some queue will appear. The participants need to define how this queue will appear and will increase with time.

The attached document provides, for each crossroad of each highway (for example I-10 Westbound), the number of incoming and outgoing vehicles per hour, as well as the flux immediately after the crossroad. The flows inside the document correspond to the situation on January 29th 2018 at 8 a.m.

For instance, on the I-10 Eastbound, at crossroad Dairy Ashford Road (second line of the attached document), the number of incoming vehicles is 1 327 vehicles per hour, the number of outgoing vehicles is 2 671 vehicles per hour. The traffic just before the crossroad is 14 112 vehicles per hour. Then the traffic just after the crossroad is $14\,112+1\,327-2\,671=12\,768$ vehicles per hour (second line of the attached document).

When there is an interconnection between two highways, the outgoing flux from the first highway to the second highway must be equal to the incoming flux on the second highway coming from the first highway: we have forced these values to be equal.

So, at the end of Phase 1, the participants should have a clear picture of the various flux on all branches of Houston's highway system, and see where traffic jam appear and how. This description is part of the Game.

In the attached file, the distances are both in miles and km.

3. Modifications in the PNR

When this is done, participants should proceed to Phase 2: modifications of the highway system. The participants may suggest any modification they consider as appropriate (such as adding new lanes, new exits, and so on), but they should:

1) Indicate what is the total cost of these modifications (they can find estimates of the costs on the Internet);
2) Indicate what is the benefit, in terms of traffic jams, of the modifications they suggest.

Here, two approaches are possible:

– The participants may determine the total duration of all trips (sum of the duration of every trip, for all vehicles), before and after the modification in the network, and they may conclude globally that each day several thousands of hours have been spared;

– Or they can choose a small number of characteristic trips, which are among the most frequented ones (for instance going from extreme West to extreme East) and determine the benefit (spare time) for these trips only.

Participants may chose the approach they prefer. It turns out that public opinion understands better the second one.

III. Participation rules

The game starts on November 1st, 2018 and ends on April 30th, 2019. Prizes will be given in May 2019, during the "Salon des Jeux Mathématiques", in Paris.

Participants should send their solution, in pdf format, in English or in French, no later than April 30th, 2019, to the email address: **ffjm@wanadoo.fr.**

No preliminary registration is required. Everyone can participate.

IV. Annex: map of Houston highway system